A DESIGN OF OPTIMAL INTEREST RATE IS ON CREDIT FOR RECEIPT OF MAXIMAL PROFIT OF COMMERCIAL BANK

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Abstract

A commercial bank that has a sufficient property asset is considered, successfully to carry out exceptionally crediting real to the sector of economy. An optimal interest rate is got on a credit that allows getting a maximal profit at the sufficient property asset of commercial bank on the end of management period. It is certain maximal income and size of sufficient property asset for realization of effective management in the conditions of vagueness.

A scientific tool that consists in formalization of suppositions in the stream model of bank is offered. The stream model of commercial bank takes into account credit activity and does not take into account delays in the terms of return of credits. They got results of research it can draw on as a model of credit activity of bank; as an example of paradox of Bertrana in bank activity. They got results of research are expedient for the maximal estimation of income and capital of bank depending on market conditions; for the analysis of market situation, when interest rates on a credit and volumes of the given out credits diminish simultaneously. They got results of research are expedient for development of models of bank activity and raising of administrative tasks, that take into account credit and deposit activity of bank, delay in the terms of return of credits, property asset of bank, a size of that can be insufficient for satisfaction of maximal demand on credits.

Key words: bank, loans, profit maximization, capital maximization, optimal control, bank’s flow model.

Introduction

One of the key functions of banks are raising funds of economic entities in deposits, placing them on his behalf and at its expense, as well as opening and maintaining bank accounts of individuals and entities. A special role is played by credit operations. A properly organized work, clearly articulated strategy and tactics of credit transactions determine both financial and competitive position of the bank under uncertainty. Central as of the position of financial forecasting, planning and management of credit operations is to develop mathematical models and programs of the bank credit, which involve not only the methodological support of their implementation, but also modelling of reserve bank, assessing income under the interest rates on loans. Significant importance assumes the application software, which allows simulation scenarios using different algorithms in conditions of uncertainty.

One approach to modelling of banks and banking activity is to use a flow model of the bank in which the financial flow is a certain amount of money per unit of time. Flow model of bank has a number of differences, namely, continuous flow in the model; funds, received by one of the input flows can be used to form the output flow of another type. That is, the money received by the bank, mixed in a single money supply, which can be used to form each of the outgoing flows in arbitrary proportions. Flow model of bank, in our view, is reasonable for considering of various problems in terms of control theory.

Purpose of the article is to determine the credit rate for commercial bank with sufficient equity to saturate the credit market to maximize profits and capital at the end of the period and the analysis of changes in this domain in accordance with the transformation of market conditions.

Research objectives: to describe the assumptions necessary to construct models of the bank to build a single-contour bank flow model without delay, on the basis of the proposed model to determine the optimal credit rate, maximum profit and maximum capital of the bank at the end of the control period, to analyze changes in the optimal interest rate, profit and capital depending on changes in market conditions.

Literature review

Concepts on the problems of banking credit complicated and differentiated simultaneously with the change of operating conditions and circulation of debt capital. Issues of lending formation in the structure of bank’s resource base in the context of managing capital are studied by Blackburn, Chambers, Felton, Garriga, Hellwig, Hodgson, Reinhart, Rogoff, Schlenhauf, Wheelock [7, 10, 12, 16, 17, 25]. Among the most well-known behaviour of the bank-monopoly model can be attributed Monti-Klein [20, 22], in which the bank operates according to the
classical microeconomic theory of monopoly. The concept of monopolistic competition was first presented in the work of Chamberlin [9]. One of the most common models of this class is Salop model, in which differentiation between products is on the basis of transportation costs.

Simulation without mediation goes back to work Benston, Bell and Murphy [2, 3]. According to them, deposits and loans are considered as output parameters of its activities and staff costs, investments, etc. - as input. Modelling including mediation, in contrast to the previous, involves accounting for substantial levels of performance of banks as financial intermediaries. At the conceptual level this approach adequately reflects the specific tasks that solve banks [3, 4, 19, 21, 23]. D. Hancock [15] offers a cost term use of financial resources that is to net losses. Model illustrating the mechanism of pools depositories, was proposed in work of Bryant [5].

Another approach to bank’s modelling - bank regarded as an agent on market of information. It is believed that any market agent that has some information and wants receive income from it, faced with two fundamental problems. First, if he tries to sell this information, the buyer can not be sure of its reliability. Second, income derived from sales information can be negligible compared to the costs of obtaining. In a situation where pricing information is public, income may be zero. This phenomenon was named Grossman-Stiglitz paradox [14]. Campbell, Kracaw [8] and Allen [1] studied this problem and formulate methods of its solution in the face of financial intermediaries.

Gorton and Pennachi in [13] drew attention to some features banking activities on transformation of assets that are treated as financing risky projects by risk-free deposits. Under adverse selection, when some agents have private information about risky projects, risk-free deposits can be used by some agents that are not informed. At the same time under the proposed model it was shown that in the appropriate economic system participation of financial intermediaries an optional and risk-free bond that are issued directly by firms, may be replaced by deposits. Diamond [11] proposed interesting theoretical model that describe how to operate the bank as an institution of delegated monitoring.

Holmström and Tirole [18] also considered a model that examines the issue of choice between direct and banking (intermediary) funding. In it factor determining the benefits of direct funding is the amount of capital owned firm. The works of Sharpe and Rajan examined the question of relationship between banks and borrowers in the dynamics [24]. A key element contained in these models was the idea that banks seeking to establish “good” relationship with borrowers to get access to information about them. Also, as in the Diamond model, it is believed that successful in the past, firms are more likely to development of his success in the future. However, it is believed that banks possess reliable information only on those firms for which they were creditors of the previous stages; otherwise the banks should make audit procedures to the unknown firms. Several authors, namely - Bhattacharya and Chiesa [6] studied the problem of ownership of information. Its essence is that firm-borrowers may face losses if for competitors become available, some private information concerning their activities. Within this context can be concluded that the bilateral relations between the bank and the borrower may be more effective than multilateral lending.

Modelling of banking in the context of control theory and using the flow model implemented Gryshyn, Ivanenko, Kapustyan, Kozak, Kuts, Osipenko, Umryk. In particular, Grishin proposes to consider bank in terms of control theory, and described some incoming and outgoing bank flows. Osipenko describes the flow model, he calls it dynamic model of bank, with linear functions of loans and deposits and sets the problem of optimal control by credit and deposit rates, provided that all deposits are issued as loans. Ivanenko describes the flow model with a linear function of loans and deposits, which takes into account the uncertainty in the volume of deposits and loans. Kapustyan proposes to use software implementation of flow model for training of bank employees, and also adds to the flow model of bank advertising costs and inflow of deposits as a result of advertising.

However, scientific studies have not elaborated with proof to some degree the maximum value of profits and capital with optimal control. Insufficient study of issues related to modelling the situation of bank reserve and the assessment of interest income based on interest rates for loans in the banking system led to the urgency of further research in this direction.

Model

Single-contour flow model of bank without delay is describing the activity of commercial banks under the following assumptions.

Assumption A1. All profit is used to increase capital.

Since all income is used to increase capital, we offer a following formula (1):
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\( x(t) = p(t), \ 0 \leq t \leq T \) \hspace{1cm} (1)

where
- \( x(t) \) – the capital of commercial banks at the moment \( t \);
- \( x(t) \) - capital gains at time \( t \);
- \( p(t) \) - profit of commercial bank at the moment \( t \);
- \( T \) - the final moment of controlling the bank.

**Assumption A2.** Bank’s only activity is lending. Because the bank deals only with credit activity, the profits consist of interest income from lending activities and is the difference between the volume of returned loans with interest, and volume of lent loans in monetary units (formula 2):

\( p(t) = K_{in}(t) - K_{out}(t), \ 0 \leq t \leq T \) \hspace{1cm} (2)

where
- \( K_{in}(t) \) – volume of returned loans with interest at the moment \( t \) in monetary units;
- \( K_{out}(t) \) – volume of lent loans at the moment \( t \).

**Assumption A3.** The volume of issued loans (in monetary units) at a given time depends on the credit rate at the moment, thus are exist reason in the credit rate controlling (formula 3):

\( K_{out}(t) = f(u_k(t)), \ 0 \leq t \leq T \) \hspace{1cm} (3)

where
- \( u_k(t) \) – credit rate at the moment \( t \).

**Assumption A4.** Relationship between total credit and lending rate is the inverse, that is, the higher the credit rate for other things being equal, the smaller the total amount of loans the bank will issue. It logically corresponds to the function of demand for loans. With higher cost of goods (credit) - fewer buyers can afford to buy it.

**Assumption A5.** The amount of issued loans is in linear dependence from credit rate.

**Assumption A6.** Bank may meet the total demand for loans.

**Assumption A7.** The credit rate is non negative. Bank do not pays the interest to creditors, thus does not execute unprofitable activities.

**Assumption A8.** There is no differentiation of credit products, credit rate is single.

**Assumption A9.** The volume of loans issued is non negative (formula 4).

\( K_{out}(t) \geq 0, \ 0 \leq t \leq T \) \hspace{1cm} (4)

Because the relationship between the volume of issued loans and credit rate is the inverse and linear, it coincides with the function of the demand for loans. It is therefore proposed to write it as (formula 5):

\( K_{out}(t) = K - b.u_k(t), \ 0 \leq t \leq T \) \hspace{1cm} (5)

where
- \( K, b \) — coefficients of linear dependence. Although \( K \) and \( b \) enter only as coefficients, but we consider it appropriate to interpret them from an economic point of view.

Thus, with zero lending rate (the minimum permissible for the bank) the amount of outstanding loan will be \( K \). Given this, this coefficient can be interpreted as an investment market capacity, the maximum amount of demand for loans (it is not unlimited). To investigate further consider the case when \( K \geq 0 \), as at \( K<0 \) the bank can not lend \( K_{out}(t)<0 \), with any credit rate, because in this case violated the assumption of A9.

From the coefficient \( b \) depends on which size will change the amount of issued loans, if you change a certain amount of credit rate. This coefficient can be interpreted as the elasticity of demand for loans. Note that to some extent, this indicator characterizes the level of competition, i.e. with increasing competition, it will be higher. Thus, the model implicitly takes into account the presence in the market other banking institutions. Assume: \( b>0 \) for the assumption A4 of the inverse form of dependence between the volume of issued loans and credit rates. The combination of these indicators is market conditions.

It is considered that the bank has enough capital to give \( K \) loans if necessary. Since deposits are not involved, the bank must have sufficient capital for any (including maximum) amount of loans (formula 6), i.e.:

\( x(t) \geq K, \ 0 \leq t \leq T \) \hspace{1cm} (6)

Volume of issued loans dependence from the credit rate is shown in Fig. 1.

Since \( x(0) = x_0 \), then \( x_0 \geq K \).

**Assumption A10.** Loans with interest returns back at the same time as issued.

**Assumption A11.** Loans with interest guaranteed returning on time and in full.
Therefore, the formula for the volume of returned loans with interest can be written as (formula 7):

\( K_{in}(t) = K_{out}(t).(1+u_k(t)), \ 0 \leq t \leq T \) \hspace{1cm} (7)

or

\( K_{in}(t) = (K - b.u_k(t)).(1+u_k(t)), \ 0 \leq t \leq T \) \hspace{1cm} (8)
Dependence of returned loans with interest rates from credit rate is presented in Fig. 2.

Thus considered three (term, return, interest) of five (term, return, interest, differentiation, collateral) general principles of lending. Now increase of the bank’s capital can be written as (9):

$$\dot{x}(t) = K \cdot u_k(t) - b \cdot u_k(t)^2,$$

(9)

Dependence of increase in capital (profit at time $t$) is shown in Fig. 3

So, the control problem is formulated as follows (formulas 10-14):

$$x(t) \rightarrow \max u_k(t)$$

(10)

$$\dot{x}(t) = K \cdot u_k(t) - b \cdot u_k(t)^2$$

(11)

$$x(0) = x_0 \geq K$$

(12)

$$u_k(t) \geq 0$$

(13)

$$0 \leq t \leq T$$

(14)

Because increase of capital does not depend on the amount of capital in the current time, the maximum capital at the end of the period achieved when maximizing increase of capital at any moment of time during the period of control. Derived from capital gains on the lending rate is (formula 15):

$$\frac{dx(t)}{du_k(t)} = K - 2 \cdot b \cdot u_k(t)$$

(15)

In a point of local extremum (maximum, because second derivative is negative), it will be zero (formula 16):

$$K - 2 \cdot b \cdot u_k(t) = 0$$

(16)

It follows that the optimal credit rate equal to the (formula 17):

$$u_k(t) = \frac{K}{2 \cdot b}$$

(17)

The maximum capital increases (or profit) (formula 18):

$$\dot{x}^*(t) = \frac{K}{4 \cdot b}$$

(18)

Then the maximum capital of the bank at the end of control is (formula 19):

$$x^*(T) = x_0 + \frac{K^2}{4 \cdot b} \cdot T$$

(19)

Let’s analyze the results. If market conditions ($K$ and $b$) do not change (which is likely in the short term), the volume of lent loans at the optimal lending rate at a time is constant and equals $K_{out}^*(t) = \frac{K}{2}$, i.e. half of the maximum amount of demand for loans; profit at optimal credit rates at a time is constant and equals $p(t)^* = \frac{K^2}{4 \cdot b}$; bank’s capital at the end of the control period at optimal lending rate is $x^*(T) = x_0 + \frac{K^2 \cdot T}{4 \cdot b}$, the bank does not change interest rates for the period of control.

If market conditions change, the optimal credit
rate determined by the same ratio \( \frac{K}{2b} \), with an increase in the maximum demand for loans \( K \) (with other things being equal) increases the optimal credit rate \( u^*_t(t) \), the optimal amount of credit granted \( K_{out}(t) \), maximal profit \( p(t) \), maximal capital of the bank at end of control period \( x'(T) \), as shown in Fig. 4.

![Fig. 4. The volume of credits issued at moment t varies depending on the change of maximal demand for loans](image)

If the maximum demand for loans decreases respectively decreases the optimal credit rate, the optimal amount of credit granted maximal profit and capital of the bank at the end of the control period. In other words, when the number of creditors is reduced, the bank had to offer more affordable interest rate.

If the elasticity of demand for loans \( b \) (the angle of the dependence line of lent loans from credit rate or the level of competition) increases, the optimal credit rate \( u^*_t(t) \), the maximal profit \( p(t) \) and maximal capital of the bank at the end of the control period \( x'(T) \) are reduced.

Conversely, if the elasticity of demand for loans is reduced (less competitive fight), the optimal credit rate, the optimal profits and capital of the bank at the end of the period for optimal control increases. The optimal amount of credit granted \( K_{out}(t) \) remains unchanged regardless of changes in elasticity of demand for loans.

Under intense competition the bank forced to reduce interest rates and vice versa - in conditions closer to the monopoly bank will increase credit rate (Fig. 5). If the elasticity of demand for loans tends to infinity, the optimal credit rate tends to zero. This repeats the result of Bertrand paradox - in a perfect competition, producers of similar goods that compete solely on price, profits will be zero, and prices of goods (credit rate) will equal the cost. Since this is our own resources (the cost of this resources is zero) credit rate (price) tends to zero.

![Fig. 5. The volume of loans issued at moment t varies depending on changes in the elasticity of demand for loans](image)

If the maximum demand for loans and the elasticity of demand for loans changes in the same number of times, the optimal credit rate will not change. Although model does not limit the maximum credit rate, it does not goes to infinity. When the maximum demand for loans is zero, there is no meaning in credit activity - the amount of credit granted will be equal to zero \( K_{out}(t) = \frac{K}{2} = \frac{0}{2} = 0 \).

Conclusions

Optimum credit rate, maximum profit and maximum capital at the end of control period for bank that has only credit activity with equity sufficient to meet maximum demand for loans and without delay in terms of repayment of loans were obtained.

Assumptions for the bank flow model with exclusively lending activities and without delays in terms of repayment of loans and the bank’s capital, sufficient to meet maximum demand for loans were formalized, model with such assumptions was constructed and analyzed, profit and capital at the end of control period maximization problem was stated and its analytic solution was obtained.

The results can be used, firstly, to illustrate the credit of the bank, secondly, to illustrate the paradox of Bertrand in banking activities, thirdly, to assess of maximum revenue and capital depending on market conditions, fourthly, to illustrate and analyze the market situation, when both credit rates, and volumes of issued loans (as in the situation of decline in production) are reduced, fifth, to
further develop banking models based on these results and to formalize optimal control problems of bank, that will take into account both credit and deposit activities of commercial banks, the delay in terms of repayment of loans and deposits, bank capital that is insufficient to meet maximal demand for loans.

References